

# Vacuum-polarization corrections to the parity-nonconserving $6s - 7s$ transition amplitude in $^{133}\text{Cs}$ .

W. R. Johnson\*, I. Bednyakov and G. Soff

*Institut für Theoretische Physik, Technische Universität Dresden, Mommenstrasse 13, D-01062  
Dresden, Germany*

## Abstract

The dominant one-loop radiative corrections to atomic wave functions, those associated with vacuum polarization in the nuclear Coulomb field, are evaluated for the  $6s - 7s$  parity-nonconserving (PNC) transition amplitude in  $^{133}\text{Cs}$ . These corrections increase the size of the PNC amplitude by 0.4% and, correspondingly, increase the difference between the experimental value of the weak charge  $Q_W(^{133}\text{Cs})$  and the value predicted by the standard model.

PACS numbers: 32.80.Ys, 11.30.Er, 31.30.Jv

Typeset using REVTeX

# I. INTRODUCTION

Parity non-conservation (PNC) in atoms, described in the standard model of the electroweak interaction by exchange of  $Z$  bosons between bound electrons and nuclear quarks, leads to nonvanishing electric-dipole matrix elements between atomic states with the same parity. The nuclear spin-independent part of PNC matrix elements (arising from the vector nucleon current) is proportional to a conserved weak charge  $Q_W$ , which is sensitive to physics beyond the standard model.

Measurements of the  $6s - 7s$  PNC amplitude in  $^{133}\text{Cs}$ , following the procedure described by Bouchiat and Bouchiat [1], were carried out to 2% accuracy by Gilbert and Wieman [2] and, more recently, to 0.3% accuracy by Wood *et al.* [3]. Following the recent measurements, there was a revival of interest in the associated atomic structure calculations of the PNC amplitude [4,5]. Indeed, Bennett and Wieman [6] analyzed differences between experimental and theoretical values of amplitudes for allowed transitions, polarizabilities, and hyperfine constants in Cs and concluded that the error in the atomic structure calculations of the PNC amplitude should be reduced from 1%, the value given in Refs. [4,5], to 0.4%. Using this revised estimate of the accuracy of the calculations, they showed that the experimental value of the weak charge of  $^{133}\text{Cs}$  differed from the standard-model value [7],

$$Q_W(^{133}\text{Cs}) = -73.09 \pm 0.03, \quad (1)$$

by  $2.3 \sigma$ . This difference between experiment and theory, being one of the two largest differences reported in the current review of particle physics [7], suggested the existence of a second neutral  $Z'$  boson [8–10].

Breit corrections to the PNC amplitude, which were ignored in [4] and underestimated in [5], were shown to decrease the size of the calculated PNC amplitude by a 0.6% by Derevianko [11]; this finding was confirmed in Refs. [12,13]. Including Breit corrections reduces the difference with the standard model to  $1.6 \sigma$  if the 0.4% error in the calculations determined in [6] is assumed or to  $0.9 \sigma$  if the more conservative 1% error given in Refs. [4,5] is assumed.

Radiative corrections to PNC matrix elements  $\langle w | H_{\text{PNC}} | v \rangle$  in the strong Coulomb field of a highly-charged one-electron ion were considered recently by Bednyakov *et al.* [14]. These corrections were decomposed into two parts, radiative corrections to the operator  $H_{\text{PNC}}$ , and radiative corrections to wave functions  $|v\rangle$  and  $|w\rangle$ . The former corrections were evaluated for  $^{133}\text{Cs}$  in Refs. [15–18] and are already included in the theoretical value  $Q_W(^{133}\text{Cs})$  given in Eq. (1). The dominant part of the wave function radiative correction, which arises from the one-loop diagrams Fig. 1, was evaluated for  $2s - 2p_{1/2}$  matrix elements in highly-charged one-electron ions in Ref. [14]. Sushkov, in his analysis of the Breit corrections [19], suggested that the residual radiative corrections to PNC amplitudes in many-electron atoms could be of the same order of magnitude as the Breit corrections. We examine this suggestion further in the present paper and find that the one-loop wave-function radiative corrections increase the size of the PNC amplitude in  $^{133}\text{Cs}$  by 0.4% and, correspondingly, increases the difference between the theoretical and experimental weak charge. Moreover, we find that the one-loop wave function correction is insensitive to electron-electron correlation effects.

## II. CALCULATION

One-loop radiative corrections (vacuum polarization corrections) in an external Coulomb field were considered by Wichmann and Kroll [20], who showed that these corrections simply modify the electron-nucleus Coulomb interaction at short range. The modification of the Coulomb interaction is described, to leading order in powers of  $Z\alpha$ , by the Uehling potential: [21]

$$\delta V(r) = -\frac{2\alpha Z}{3\pi r} \int_1^\infty dt \sqrt{t^2 - 1} \left( \frac{1}{t^2} + \frac{1}{2t^4} \right) e^{-2ctr}. \quad (2)$$

We use atomic units here;  $\alpha = 1/137.036\dots$  is the fine structure constant and  $c \equiv \alpha^{-1}$  is the speed of light.

In applications to atomic PNC, the Coulomb potential is also modified at short range by finite nuclear size effects, which are described by a Fermi-type charge distribution  $\rho(r)$ :

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - c_{\text{nuc}})/a_{\text{nuc}}]}. \quad (3)$$

For  $^{133}\text{Cs}$ , we assume that the central radius  $c_{\text{nuc}} = 5.6748$  fm and that the 10%–90% falloff distance is  $t_{\text{nuc}} = 2.3$  fm, corresponding to  $a_{\text{nuc}} = 0.523$  fm and to a root-mean-square radius of the nuclear charge distribution  $R_{\text{rms}} = 4.807$  fm. As shown by Fullerton and Rinker [22], the Uehling potential can be generalized as follows to accommodate the charge distribution  $\rho(r)$  of an extended nucleus:

$$\delta V(r) = -\frac{2\alpha^2}{3r} \int_0^\infty dx x \rho(x) \int_1^\infty dt \sqrt{t^2 - 1} \times \left( \frac{1}{t^3} + \frac{1}{2t^5} \right) \left( e^{-2ct|r-x|} - e^{-2ct(r+x)} \right). \quad (4)$$

Corrections to Dirac-Coulomb energies for  $1s$ ,  $2s$  and  $2p$  electrons in one-electron ions obtained by solving the Dirac equation in the composite nuclear + Uehling potential discussed above are found to be in close agreement with sums of the Uehling, Uehling-finite nuclear size, and finite nuclear size corrections obtained perturbatively in [23]. A comparison of the present results with those from [23] are given in Table I. Uehling corrections to  $ns$  and  $np_{1/2}$  levels of Cs (the levels of primary interest in PNC calculations in Cs) are about twice as large as finite nuclear size corrections and have opposite signs as shown in the table for levels with  $n=1$  and 2.

We carry out two calculations of the  $6s - 7s$  PNC amplitude in the modified potential, both leading to precisely the same relative correction to the PNC amplitude. The first of these calculations, is done at the “weak” Dirac-Hartree-Fock (DHF) level of approximation. The perturbation  $\delta\phi_v^{\text{HF}}$  to a valence electron wave function  $\phi_v^{\text{HF}}$  induced by the *weak* interaction  $h_{\text{PNC}}$  satisfies the inhomogeneous DHF equation

$$(h_0 + V^{\text{HF}} - \epsilon_v^{\text{HF}}) \delta\phi_v^{\text{HF}} = -h_{\text{PNC}}\phi_v^{\text{HF}}. \quad (5)$$

In this equation,  $V_{\text{HF}}$  is the HF potential of the closed xenon-like core and  $\epsilon_v^{\text{HF}}$  is the eigenvalue of the unperturbed DHF equation. The perturbed DHF equations are solved to give the  $\delta\phi_{6s}^{\text{HF}}$  and  $\delta\phi_{7s}^{\text{HF}}$ . The PNC amplitude is then given by the sum of two terms:

$$E_{\text{PNC}} = \langle \phi_{7s}^{\text{HF}} | D | \delta \phi_{6s}^{\text{HF}} \rangle + \langle \delta \phi_{7s}^{\text{HF}} | D | \phi_{6s}^{\text{HF}} \rangle, \quad (6)$$

where  $D$  is the dipole operator. This calculation leads to a value of the PNC amplitude that is 20% lower than the final correlated value found in Refs. [4,5,13]. In the top panel of Table II, we show each of the two terms making up the sum in Eq. (6) determined with and without the modified Uehling potential  $\delta V$  from Eq. (4). The one-loop correction is seen to increase the size of each terms and their sum by 0.41%.

The second calculation is done at the “weak” random-phase approximation (RPA) level of approximation, in which the class of correlation corrections associated with weak perturbations of the core orbitals are included in the calculation. This calculation leads to a value of the PNC amplitude that is 3% larger than the final correlated value of the amplitude given in [4,5,13]. This class of correlation corrections is included by solving

$$(h_0 + V^{\text{HF}} - \epsilon_v^{\text{HF}}) \delta \phi_v^{\text{RPA}} = -[h_{\text{PNC}} + V_{\text{PNC}}^{\text{HF}}] \phi_v^{\text{HF}}, \quad (7)$$

where  $V_{\text{PNC}}^{\text{HF}}$  is the weak correction to the HF potential. The resulting PNC amplitude is given by

$$E_{\text{PNC}} = \langle \phi_{7s}^{\text{HF}} | D | \delta \phi_{6s}^{\text{RPA}} \rangle + \langle \delta \phi_{7s}^{\text{RPA}} | D | \phi_{6s}^{\text{HF}} \rangle. \quad (8)$$

In the lower panel of Table II, we give values of the two terms making up the sum in Eq. (8) with and without the Uehling corrections. Again, the one-loop correction is seen to increase the size of each of the two terms and their sum by 0.41%.

From the two calculations above, it is apparent that the short range vacuum-polarization corrections are independent of electron-electron correlation. The situation is similar to that found for the short range nuclear “skin” correction arising from the difference between the neutron and proton radius of the nucleus [24,25], which also leads to a correlation-independent correction to the PNC amplitude.

### III. CONCLUSION

One-loop radiative corrections to the electron wave functions from vacuum-polarization in the nuclear field are evaluated. The resulting wave functions are used to calculate the PNC amplitude for the  $6s - 7s$  PNC transition in  $^{133}\text{Cs}$ , leading to a 0.4% increase the size of the amplitude. This correction is found to be correlation independent.

An average of the three most accurate calculations [4,5,13], of the  $6s - 7s$  amplitude, taking account of Breit corrections and one-loop radiative corrections gives

$$E_{\text{PNC}} = -9.057 \pm 0.037 \text{ } iea_0 \times 10^{-12} (-Q_W/N),$$

where we use the estimate from [6] for the error in the calculations. Combining this calculated amplitude with the experimental value of the amplitude from [3] leads to an experimental weak charge

$$Q_W^{\text{expt}}(^{133}\text{Cs}) = -72.12 \pm (0.28)_{\text{expt}} \pm (0.34)_{\text{theor}}$$

This value differs by  $2.2 \sigma$  from the standard model. If we assume a 1% error in the theoretical amplitude, then the theoretical component of the error in  $Q_W^{\text{expt}}$  is increased to  $\pm(0.74)_{\text{theor}}$  and the difference with the standard model becomes  $1.2 \sigma$ .

We have considered only the dominant one-loop wave-function radiative corrections in the above calculation and ignored the higher-order  $\alpha Z$  corrections to the Uehling potential discussed by Wichmann and Kroll [20]. The Wichmann-Kroll corrections can be easily estimated and are expected to have a negligible effect on the present result. Moreover, the “vertex” radiative correction to the PNC amplitude, which contributes about 0.1% to the Standard Model value of  $Q_W$ , is evaluated in Refs. [15,17] using free-particle propagators. This radiative correction should be redone using Coulomb-field propagators. If Coulomb-field effects were to change the vertex correction by 100% (probably a gross overestimate) the Standard Model value of  $Q_W$  would change by only 0.1%. We therefore expect that further changes to the value of  $Q_W$  from the strong-field corrections to the operator  $H_{\text{PNC}}$  discussed in [14] will be smaller than 0.1%.

## ACKNOWLEDGMENTS

The work of WRJ was supported in part by an Alexander von Humboldt award from the Federal Republic of Germany and in part by the U. S. National Science Foundation under grant No. 99-70666. WRJ owes a debt of gratitude to O. Sushkov for helpful discussions. GS and IB acknowledge fruitful discussions with L. Labzowsky and support by the BMBF and GSI (Darmstadt).

# FIGURES

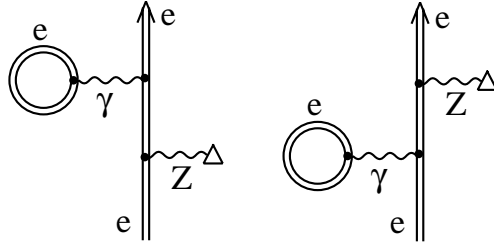


FIG. 1. One loop wave function corrections. The double line represents the electron in the field of the nucleus, the wavy lines represent  $Z$  bosons or photons and the open triangle represents the  $Z$ -nucleus vertex.

# TABLES

TABLE I. Differences between the present one-electron Dirac energies for Cs ( $Z=55$ ) in the finite nucleus plus Uehling potential and Dirac-Coulomb energies ( $\delta E$ ) are compared with perturbative values of Uehling (Uehl), Uehling-finite nuclear size (Uehl-FS), and finite nuclear size (FS) corrections to one-electron energy levels given in [23]. Units:  $\alpha mc^2(\alpha Z)^4/(\pi n^3)$

State	Present	Ref. [23]			
	$\delta E$	Uehl	Uehl-FS	FS	Total
$1s_{1/2}$	-0.1419	-0.2584	0.0010(1)	0.1159	-0.1415(1)
$2s_{1/2}$	-0.1554	-0.2901	0.0011(1)	0.1339(1)	-0.1551(2)
$2p_{1/2}$	-0.0100	-0.0145	0.0000(1)	0.0045	-0.0100(1)
$2p_{3/2}$	-0.0016	-0.0016	0.0000	0.0000	-0.0016

TABLE II. One-loop corrections to the weak HF-PNC amplitude for the  $6s-7s$  PNC amplitude for  $^{133}\text{Cs}$  are shown in the upper panel and to the weak RPA-PNC amplitude in the lower panels. Values are listed without and with corrections from Eq. (4). Units:  $iea_0 \times 10^{-12}(-Q_W/N)$

Type	$\langle \phi_{7s}   D   \delta \phi_{6s} \rangle$	$\langle \delta \phi_{7s}   D   \phi_{6s} \rangle$	$E_{\text{PNC}}$
Weak HF approximation:			
DHF	2.749183	-1.014387	-7.394685
DHF+ $\delta V$	2.760414	-1.018581	-7.425391
$\Delta$ (%)	0.41	0.41	0.41
Weak RPA approximation:			
RPA	3.457036	-1.272562	-9.268581
RPA+ $\delta V$	3.471169	-1.277834	-9.307166
$\Delta$ (%)	0.41	0.41	0.41

## REFERENCES

- \* Permanent Address: Department of Physics, 225 Nieuwland Science Hall, University of Notre Dame, Notre Dame, IN 46556 U. S. A.
- [1] M. A. Bouchiat and C. Bouchiat, J. Phys. (Paris) **35**, 899 (1974); **36**, 493 (1974).
  - [2] S. L. Gilbert and C. E. Wieman, Phys. Rev. A **34**, 792 (1986).
  - [3] C. S. Wood *et al.*, Science **275**, 1759 (1997).
  - [4] V. A. Dzuba, V. V. Flambaum, O. P. Sushkov, Phys. Lett. A **141**, 147 (1989).
  - [5] S. A. Blundell, J. Sapirstein, and W. R. Johnson, Phys. Rev. D **45**, 1602 (1992).
  - [6] S. C. Bennett and C. E. Wieman, Phys. Rev. Lett. **82**, 2484 (1999); **82**, 4153 (1999); **83**, 889 (1999).
  - [7] D. E. Groom et al., Euro. Phys. J. C **15**, 1 (2000).
  - [8] R. Casalbuoni, S. De Curtis, D. Dominici, R. Gatto, Phys. Letts. B **460**, 135 (1999).
  - [9] J. L. Rosner, Phys. Rev. D **61**, 016006 (1999).
  - [10] J. Erler and P. Langacker, Phys. Rev. Lett. **84**, 212 (2000).
  - [11] A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000).
  - [12] V. A. Dzuba, C. Harabati, W. R. Johnson, and M. S. Safronova, Phys. Rev. A **63**, 044103 (2001).
  - [13] M. G. Kozlov, S. G. Porsev, and I. I. Tupitsyn, Phys. Rev. Lett. **86**, 3260 (2001).
  - [14] I. Bednyakov, L. Labzowsky, G. Plunien, G. Soff and V. Karasiev, Phys. Rev. A **61**, 012103 (1999).
  - [15] B. W. Lynn and P. G. H. Sandars, J. Phys. B **27**, 1469 (1994).
  - [16] W. J. Marciano and J. L. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); **68**, 898 (1992).
  - [17] W. J. Marciano and A. Sirlin, Phys. Rev. D **27**, 552 (1983).
  - [18] A. Sirlin, Phys. Rev. D **22**, 971 (1980).
  - [19] O. P. Sushkov, Phys. Rev. A **63**, 042504 (2001).
  - [20] E. H. Wichmann and N. H. Kroll, Phys. Rev. **101**, 843 (1956).
  - [21] E. A. Uehling, Phys. Rev. **48**, 55 (1935).
  - [22] L. W. Fullerton and G. A. Rinker, Jr., Phys. Rev. A **13**, 1283 (1976).
  - [23] W. R. Johnson and G. Soff, At. Data Nucl. Data Tables **33**, 405 (1965).
  - [24] S. J. Pollock and M. C. Welliver, Nucl. Phys. A **663**, 381c (2000); J. Phys. G **27**, 787 (2001).
  - [25] J. James and P. G. H. Sandars, J. Phys. B **32**, 3295 (1999).